A (Historical) Review of the Six-Port Measurement Technique

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***Abstract*—The six-port measurement technique has found wide acceptance and has played a major role in microwave metrology, particularly in national standards laboratories. This paper reviews the development of this technology and includes some historical background.**

***Index Terms*—Automatic network analyzer, microwave measurements, microwave metrology, six-port, vector automatic network analyzer.**

I. Background

THE FIELD OF microwave metrology differs from its lower frequency counterpart in that a uniform transmission line or waveguide is required in order that the fields therein may be described as a pair of well-defined traveling waves (in the forward and reverse directions) and whose *complex* amplitudes will be denoted by a and b. These wave amplitudes provide the basis for microwave circuit theory and their measurement is a major objective of microwave metrology. Unfortunately, any attempt to observe these waves (e.g., via probes, etc.) at the position of interest also violates the uniformity requirement. Instead, one infers these amplitudes at the "test port" or terminal surface from observations at other locations.

This may be illustrated as in Fig. 1, which includes a source, which is connected to the multiport junction, a test port (#2), and detectors at the one or more remaining ports. The operation is governed by one or more equations of the form

$$b_3 = Aa_2 + Bb_2 \tag{1}$$

where b_3 is the complex signal provided to the sensing device, a_2, b_2 are the complex wave amplitudes at the terminal of interest, and A, B are parameters which characterize the measuring instrument. In an environment where the sensing device responds only to power, one has

$$|b_3|^2 = |Aa_2 + Bb_2|^2. \tag{2}$$

From the perspective of a microwave metrologist, it is possible to recognize at least three different eras in the evolution of the art. Much of the early development was, of course, an outgrowth of the technology developed during the second World War. In its aftermath, volume 11 of the Radiation Laboratory Series [1] was a major reference work. This era was

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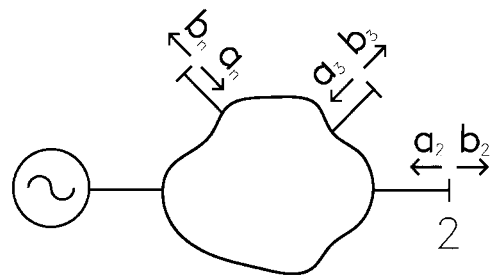

Fig. 1. Microwave metrology is characterized by the use of detectors which are "remote" from the terminal surface of interest.

characterized by a heavy dependence upon simple detection schemes using detectors of the diode or bolometric type. During this time the "key" to improved accuracy was frequently an improved item of hardware. (For example, an improved probe transport mechanism for a standing wave machine, or a directional coupler of improved directivity.) In terms of (1), a standing wave machine is a device for which $|A| = |B|$ while the phase difference is adjustable via the probe position. For the ideal directional coupler, either A or B would vanish. In order to support this requirement, an instrument shop and a highly trained staff of instrument makers accompanied the National Bureau of Standards (NBS) Microwave Metrology Section when it was moved from Washington, DC, to Boulder, CO, in 1954.

The second era was characterized by the substitution of the directional coupler and "reflectometer" for the standing wave machine. Moreover, by the incorporation of "tuning transformers" therein, it was possible make *in situ* adjustments of their parameters ($A = 0$ or $B = 0$) and thus obtain improved directivity, etc. In time this became a highly developed technology and served the art rather well for a decade or more. On the other hand, these methods were both frequency sensitive and time consuming. With the advent of the digital computer, which made possible the (vector) automated network analyzer (VANA), these methods were headed for obsolescence, and the third era was introduced.

II. Introduction

In today's world the VANA can accomplish in seconds, if not milliseconds, measurement results which formerly would have required a day or more. In addition to the speed and operator convenience, however, a major shift in "measurement strategy" is associated therewith. In particular, the "requirement" for reduced mechanical tolerances in the hardware, or time-consuming tuning adjustments, the techniques include *an explicit recognition (or modeling) of the hardware imperfections and their "elimination" by the software*.

A key element in the implementation of this new strategy was a detection system which responded to the *phase* as well as amplitude of the detected signals. Returning to Fig. 1, a second detector is added whose operation may be described by

$$b_4 = Ca_2 + Db_2. \tag{3}$$

Given this response, and the parameters $A \cdots D$, one has a system which may be solved for a_2 and b_2. (As a practical matter, the primary interest is usually in the ratio a_2/b_2, for which only b_3/b_4, A/D, B/D, and C/D are required.) These may be determined by a "calibration procedure" which typically requires a set of "known" terminations, or perhaps a length of transmission line, and the VANA response thereto.

Although the major portion of the six-port development has taken place in the last of these eras, this historical review would not be complete without acknowledging some of the earlier work. Of note is a paper by Samuel [2] who described an "oscillographic" method of presenting impedance. Although the term had not been coined at that time, his "six-port" took the form of a length of waveguide with four probes inserted therein. Because the appropriate probe spacing was frequency dependent, this was a narrow-band technique. In addition, Samuel's paper recognized the potential application of directional couplers to this problem. This work was followed some decades later by that of Cohn [3] and Hoer [4], who described systems along these lines. Although these provided improved bandwidth, the resultant accuracy was still restricted by the deviations of the hardware from its design objectives.

Perhaps the defining moment in the emergence of the six-port technology as it exists today was the recognition that an explicit correction could be made for these deviations in a six-port network while retaining the simple amplitude detection methods [5], [6]. This was in marked contrast to the VANA systems, then under development within the industry, which required heterodyne detection and its associated frequency conversion, local oscillators, etc.

The operation of the six-port may be defined by the system of equations

$$|b_3|^2 = P_3 = |Aa_2 + Bb_2|^2 \tag{4}$$
$$|b_4|^2 = P_4 = |Ca_2 + Db_2|^2 \tag{5}$$
$$|b_5|^2 = P_5 = |Ea_2 + Fb_2|^2 \tag{6}$$
$$|b_6|^2 = P_6 = |Ga_2 + Hb_2|^2 \tag{7}$$

where $P_3 \cdots P_6$ are the sidearm responses, b_2 and a_2 are, respectively, the emergent and incident wave amplitudes at the test port, and $A \cdots H$ are the six-port network parameters.

Expanding (4), one has

$$P_3 = |A|^2|a_2|^2 + AB^* a_2 b_2^* + A^* B a_2^* b_2 + |B|^2|b_2|^2 \tag{8}$$

while similar expansions of (5)–(7) are also possible. By inspection, (8) is "linear" in $|a_2|^2$, $a_2 b_2^*$, $a_2^* b_2$, and $|b_2|^2$, thus the solution of (4)–(7) for these "unknowns" is linear in $P_3 \cdots P_6$. Moreover, the *difference* between $|b_2|^2$ and $|a_2|^2$, which is equal to the net power (P_{net}) at the test port, will also be linear in $P_3 \cdots P_6$. The reflection coefficient Γ_l may be written

$$\Gamma_l = \frac{a_2}{b_2} = \frac{a_2 b_2^*}{|b_2|^2} \tag{9}$$

so that this may be obtained as well.

This elementary theory, however, did not account for certain redundancies. In particular, one has *four* observations in (4)–(7) from which to determine P_{net} and the *complex* Γ_l. In time it was demonstrated that the four power meter readings are connected by a quadratic equation such that three of them determine the fourth to the extent of a choice between two possible values. This made it possible to both improve and assess the measurement accuracy, and it led to simplified calibration procedures (i.e., a determination of the parameters which characterize the six-port).

III. General Theory

Although the details of the more general solution to (4)–(7) are beyond the scope of this paper, a general outline is as follows. For initial convenience it is assumed that, in keeping with the usual design objectives, $C = 0$. Then (4) and (5) may be combined to obtain

$$|\Gamma_l - q_3|^2 = \frac{|D|^2}{|A|^2} \cdot \frac{P_3}{P_4} \tag{10}$$

where $q_3 = -B/A$. By hypothesis A, D, and q_3 are known, and P_3/P_4 is observed so that Γ_l lies on a circle with center at q_3 and radius as given by the right-hand side of (10). In a similar way, (6) and (7) may be combined with (5) to obtain two additional circles whose centers will be denoted by q_5 and q_6. The solution may be thus indicated graphically, as in Fig. 2. In the general case, where $C \neq 0$, the circle centers as well as the radii are functions of the power meter readings. This geometric picture has provided a substantial amount of intuitive insight into the six-port operation [7]. In particular, two of the circles determine the radius of the third to the extent of a choice between two possible values. Thus, and as noted earlier, there is a quadratic relationship among the P_i. To be more explicit, the ratios P_3/P_4, P_5/P_4, and P_6/P_4 lie on a paraboloid surface in a three-dimensional "P-space."

The analytic solution may be simplified by introducing a two-step procedure in which the first objective is to determine the *complex* ratio b_3/b_4, which will be denoted by w. Following this, conventional four-port reflectometer theory [8] may be used. (In six-port parlance, this initial step is known as a six-to-four-port reduction.) Starting with (1) and (3), one can solve for a_2, b_2 and then substitute in (6) and (7). This leads to

$$|w|^2 = P_3/P_4 \tag{11}$$
$$|w - w_1|^2 = gP_5/P_4 \tag{12}$$

and

$$|w - w_2|^2 = hP_6/P_4 \tag{13}$$

where w_1, w_2, g, and h are functions of $A \cdots H$. Again the solution, which is now "exact," is given by the intersection

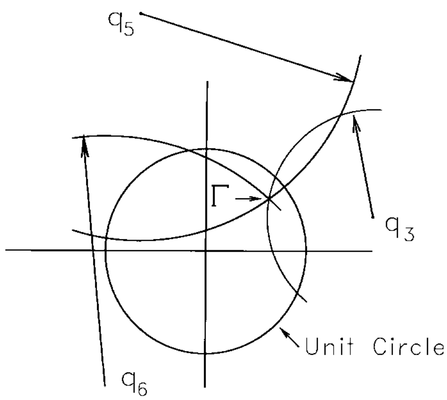

Fig. 2. The reflection coefficient is determined from the intersection of three circles.

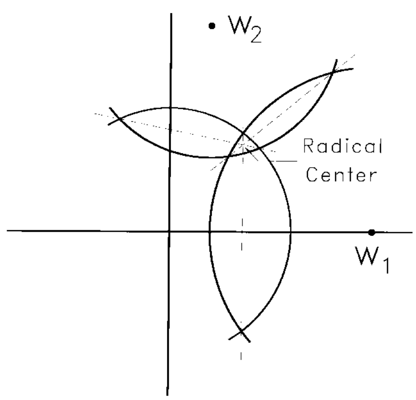

Fig. 3. The "intersection failure" is an indicator of power meter error.

of three circles centered at the origin w_1 and w_2. Moreover, since only phase *differences* are involved, it is both possible and convenient to assume that w_1 is real. In practice, because of measurement error in the P_i, the circles will not intersect in a point. The situation is thus as given in Fig. 3, where the "intersection failure" is an indication of the power meter error and provides a useful performance monitor [9], [10]. With reference to the "linear" solution, which was introduced in an earlier paragraph, it can be shown that this yields the intersection of the common chords, or "radical center," as shown in Fig. 3. As noted above, once w has been determined, the problem reduces to conventional four-port reflectometer theory in which w is related to Γ_l by a linear fractional transform.

IV. Dual Six-Port

In order to measure two-port devices, the *dual* six-port was developed [11]. As shown in Fig. 4, this is implemented by a pair of six-ports which are fed from a common source and dividing network. The latter includes a provision for adjusting the phase difference between the signal inputs to the two six-ports. In operation, the "reflection coefficient" as observed by six-port 1, at terminal 1, is augmented by the signal which is fed through the device under test (DUT) via six-port 2. This measurement is repeated for several different phase differences, which makes it possible to distinguish between the transmitted signal and the actual reflection from the DUT.

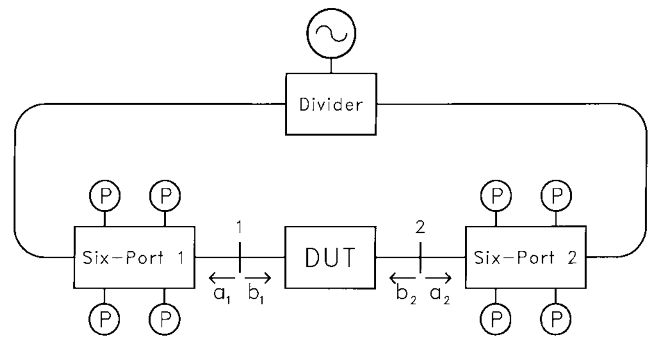

Fig. 4. The dual six-port.

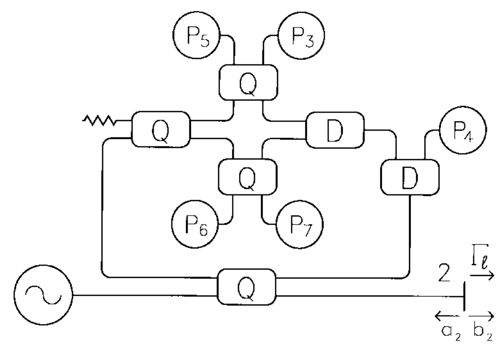

Fig. 5. A typical six-port circuit.

For a 40-dB attenuator, the transmitted signal is of the same amplitude as that provided by a reflection from the DUT of 0.01. The dynamic range in an attenuation measurement is thus limited by the resolution of the power detectors in use rather than their dynamic range. For these reasons, the detectors of choice are of the bolometric type, although extensive use has also been made of diode types in less demanding applications.

V. Calibration Methods

The calibration requires the determination of two sets of parameters: those associated with the six-to-four-port reduction, and the three complex ones which characterize the w to Γ_l transform. As noted above, there is a quadratic relationship among the P_i, which is characterized by five parameters, from which one may obtain the w_1, w_2, g, and h. These parameters may be determined by observing the system response to a collection of terminations, but whose values need not be known except as required to obtain a well-conditioned solution. The parameters which characterize the w to Γ_l transform may be obtained by any of the existing techniques for a four-port reflectometer. For the dual six-port, the "thru-reflect-line" (TRL) [12] and its variants are convenient.

VI. Six-Port Circuits

The design for the six-port network revolves primarily around the choice of positions for the circle centers. From symmetry, these should be equidistant from the origin and spaced at 120°. The optimal distance from the origin is problematic, but a value of 1.5 is satisfactory in most applications. The

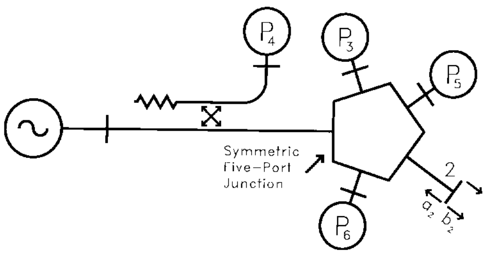

Fig. 6. Six-port circuit based on the use of a symmetric five-port.

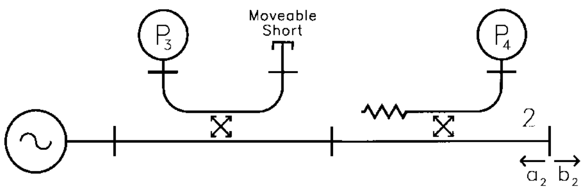

Fig. 7. The multistate reflectometer is a variant of the six-port.

six-port may be assembled from quadrature (Q) and divider (D) hybrids, and the circuit shown in Fig. 5 is typical. In reality, this is a seven-port, and if the meter P_7 is replaced by a termination, the spacing for the circle centers is 90°, 90°, and 180°. Somewhat better spacing (90°, 135°, and 135°) may be achieved via an alternative circuit which uses fewer hybrids [13]. Another circuit is shown in Fig. 6 [14].

An interesting variant of the six-port is provided by the "multistate reflectometer," the circuit for which is shown in Fig. 7. Here there are only two detectors, but P_3 is observed for three or more positions of the sliding short. For additional details the reader is referred to the original paper [15].

ACKNOWLEDGMENT

Because of space limitations and other constraints, it has only been possible to identify some of the major milestones in the emergence of the six-port art, with a primary focus on those with which the author is the most familiar. Unfortunately, there is no way the author can acknowledge all of the many contributions to its development. Of primary note, however, is that of the author's colleague, C. Hoer, whose insight into its potential applications and other contributions provided a substantial stimulus in the early development. Among the author's colleagues at NIST, J. Juroshek (and his supporting staff) warrants attention both for his contributions to the implementation of the six-port systems as they currently exist, and for their maintenance over the years that have followed the author's retirement from NIST. From here the circle of contributors extends to the national primary laboratories and universities in England, Germany, France, Canada, Denmark, Korea, China, and elsewhere. To the many of you who have contributed to this development, the author extends his appreciation and the hope that we can all meet in that "better land."

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Dr. Engen's contributions to microwave metrology have been recognized by the Department of Commerce Silver and Gold Medals, the National Bureau of Standards Applied Research Award, the (Instrumentation and Measurement) Society Award, the Woodington Award for Professionalism in Metrology, and the IEEE Automatic RF Techniques Group Automated Measurement Career Award.